CONVECTIVE HEAT EXCHANGE OF A VISCOPLASTIC FLUID WITH TEMPERATURE-DEPENDENT RHEOLOGICAL CHARACTERISTICS DURING STRUCTURAL FLOW IN A CIRCULAR PIPE

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Heat exchange in a viscoplastic liquid moving in a circular pipe is investigated, taking into account the dependence of plastic viscosity and ultimate shear stress on temperature. A system of motion, energy, and continuity equations transformed under the assumption that the Pe and Pr numbers are much greater than 1 is solved on a computer by the method of finite differences using iterations. Results of the numerical solutions for the exponential form of the dependences of the rheological characteristics on temperature are analyzed in detail. A comparison of the numerical solutions with well-known theoretical solutions in particular cases and also with experimental data indicates their high precision.

Many theoretical solutions of convective heat-transfer problems in Newtonian and non-Newtonian fluids carried out by assuming that the fluid properties are constant are known [1-7].

The problem of taking into account the influence of the temperature dependence of the rheological characteristics of a fluid on flow and heat transfer has recently drawn even greater interest. This problem has been investigated for a Newtonian [2, 8] and a non-Newtonian fluid with exponential rheological equations [9-11]. Recently-published works [12, 13] have considered the convective heat-exchange problem in a fluid that obeys the rheological Buckley–Herschel equation. It has been proposed [12] that ultimate shear stress is constant and that consistency depends on temperature according to a hyperbolic law, and that radial convective heat transfer is negligibly small. The work [13] is one which is free of the restrictive assumptions made in [12]. However, the specific feature of the problem, namely, the presence of a flow core, was ignored here, this presence implying, in particular, that the solution found this way will not agree with well-known solutions of the problem in actual cases [3-7]. Concrete results of the solution of this problem are lacking in [13].

In this work, the influence of the temperature dependence of rheological characteristics on flow and heat transfer is investigated for the example of convective heat exchange in a viscoplastic fluid that obeys the Shvedov–Bingham equation. The latter is a particular case of the Buckley–Herschel equation and is used as the most common approximation of the rheological behavior of paraffin-base and resinous petroleum and petroleum products in a broad temperature range.

Let us consider steady-state structural flow of a viscoplastic fluid in a circular pipe of radius R induced by the effect of a longitudinal pressure drop.

A constant temperature T_W is maintained on the pipe wall and the fluid temperature at the pipe input is uniformly distributed and equal to T_0, $T_W \neq T_0$.

Suppose the rheological characteristics of a viscoplastic fluid, the plastic viscosity μ, and ultimate shear stress τ_0 depend on temperature T, and that the density, heat conductivity, and heat capacity are constant. We assume, moreover, that the fluid is incompressible:

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$$\mu(T)=\mu_0 f_1(T), \quad \tau_0(T)=\tau_{00} f_2(T) \tag{1}$$

In Eq. (1) μ_0 and τ_{00} are plastic viscosity and ultimate shear stress at temperature T_0, respectively.

Our problem under the condition that mass forces can be neglected corresponds to the system of equations

$$\begin{gathered}\rho(\mathbf{v}\nabla)\mathbf{v}=\operatorname{div}\Pi; \\ \Pi=2[\mu(T)+1/h\tau_0(T)]\dot{S}-p\mathscr{E}; \\ \mathbf{v}\operatorname{grad} T=a\{\nabla^2 T+1/k\; J[\mu(T)h+\tau_0(T)]h\}; \\ \operatorname{div}\mathbf{v}=0,\end{gathered} \tag{2}$$

where Π and S are the stress tensor and deformation velocity tensor, ξ is a unit tensor, h is the degree of sliding deformation velocity, **v** is the velocity vector, p is pressure, ρ is density, a and k are the thermal diffusivity and heat transfer, respectively, of the fluid, and J is the mechanical equivalent of heat.

The first two equations in the system (2) are the motion equations for a viscoplastic Henke–Il'yushin medium, the third equation is a heat-transfer equation is which the dissipative heat-release term is taken into account for generality [3], and the fourth equation is a continuity equation.

Let us introduce a cylindrical coordinate system whose z axis is directed along the pipe axis. Bearing in mind axial symmetry, we find that the tangential velocity component v_φ and the derivatives of all the variables with respect to φ vanish. In this case, the system (2) has the component-wise form

$$\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial r}\left[2(\mu+1/h\tau_0)\frac{\partial v_r}{\partial z}\right]+\frac{\partial}{\partial z}\left[(\mu+1/h\tau_0)\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right]+\frac{2}{r}(\mu+1/h\tau_0)\left(\frac{\partial v_r}{\partial r}-\frac{v_r}{r}\right); \tag{3}$$

$$\rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}\left[(\mu+1/h\tau_0)\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left[2(\mu+1/h\tau_0)\frac{\partial v_z}{\partial z}\right]+1/r(\mu+1/h\tau_0)\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right);$$

$$v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}=a\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+1/kJ(\mu h+\tau_0)h\right];$$

$$\frac{\partial(rv_r)}{\partial r}+\frac{\partial(rv_z)}{\partial z}=0;$$

$$h\equiv\left[\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)^2+2\left(\frac{\partial v_r}{\partial r}\right)^2+2\left(\frac{v_r}{r}\right)^2+2\left(\frac{\partial v_z}{\partial z}\right)^2\right]^{1/2}.$$

Here r is the current radius and v_z and v_r are the longitudinal and transverse velocity components, respectively.

Significant difficulties are encountered in the solution of the system (3), even with the use of numerical methods on a computer.

A natural method for overcoming them is to pass to correspondingly selected dimensionless variables and then to discard small terms in the transformed equations.

In selecting the dimensionless variables, we will draw on physical concepts. Convective heat exchange in pipes develops substantially differently in the longitudinal as compared to the transverse direction. In the longitudinal directions convective heat transfer occurs, while in the transverse direction molecular heat transfer occurs. The Peclet number $Pe = 2\langle v\rangle R/a$ (where $\langle v\rangle$ is the mean flow rate defined by the ratio of the volume flow rate of the fluid to the cross-sectional area of the pipe) is a measure of the ratio of convective heat transfer to heat arising in the flow due to heat conductivity. Therefore there exist two distinct physical scales of length, namely, the transverse R and the longitudinal, proportional to Pe.

Let us take as the longitudinal scale of length the value $L = PeR/2$ and as the characteristic pressure, the pressure drop at distance L for isothermic flow of a viscoplastic fluid in a circular pipe,

$$p_0=\lambda_0\rho\langle v\rangle^2/2L/2R,$$

where λ_0 is specific hydraulic resistance.

We introduce the dimensionless variables

$$\eta = r/R;\quad \xi = z\Big/\frac{R}{2}\,\mathrm{Pe};\quad P = p/p_0;\quad \theta = (T - T_W)/(T_0 - T_W);$$

$$V_z = v_z/\langle v\rangle;\ V_r = v_r/\langle v\rangle;\ \mathrm{Re} = 2R\langle v\rangle\rho/\mu_0;$$

$$= 2\tau_{00}R/\langle v\rangle\mu_0;\ \alpha = \langle v\rangle^2\mu_0/2\kappa J(T_0 - T_W),$$

where Re is Reynolds parameter, I is the Saint Venant–Il'yushin parameter, and α is a dissipation parameter.

Specific hydraulic resistance depends on the Re and I parameters [14]:

$$\lambda_0 = 64/\mathrm{Re}\varphi(\mathrm{I}).$$

We transform the system (3) to new variables with a simultaneous bound on the order of each term of the equation. Here we substantially use the fact that for most cases of structural flows of viscoplastic fluids in pipes, the Pe number reaches extremely high values, up to several thousands and even hundreds of thousands, while the Prandtl number Pr (Pr = $\mu_0/a\rho$) varies from several hundred to thousands and above. Therefore the terms of the transformed equations with factors Pe^{-n} and Pr^{-n} ($n \geq 1$) are estimated by the order of their magnitudes.

Discarding small terms in Eqs. (3) we arrive at the equations

$$\begin{gathered} dP/d\eta = 0 \quad \text{or} \quad P = P(\xi), \\ \frac{dP}{d\xi} = \frac{1}{8\varphi(\mathrm{I})}\frac{1}{\eta}\frac{\partial}{\partial\eta}\left\{\eta\left[F_1(\theta)\frac{\partial V_z}{\partial\eta} - \frac{\mathrm{I}}{2}F_2(\theta)\right]\right\}; \\ V_r'\frac{\partial\theta}{\partial\eta} + V_z\frac{\partial\theta}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right) + 2\alpha\left[F_1(\theta)\frac{\partial V_z}{\partial\eta} - \frac{\mathrm{I}}{2}F_2(\theta)\right]\frac{\partial V_z}{\partial\eta}; \\ \frac{\partial(\eta V_r')}{\partial\eta} + \frac{\partial(\eta V_z)}{\partial\xi} = 0. \end{gathered} \tag{4}$$

Here we have introduced the notation $V_r' = \mathrm{Pe}/2V_r$; and $F_{1,2}(\theta) = f_{1,2}(T)$. The order of the radial velocity and and its derivative with respect to η are estimated from the continuity equation in deriving Eqs. (4).

We note that terms containing $\partial V_z/\partial\eta$ have dropped out in the transformed motion equations. Physically, this means that the velocity profile "readjusts" as the temperature profile varies and that the initial velocity distribution in the section $\xi = 0$ does not substantially affect the development of the velocity profile along the flow, i.e., we have not taken into account in this treatment the effects due to the development of velocity in the initial hydrodynamic segment.

Let us pass to a discussion of the boundary conditions. The presence of a flow core adjacent to the longitudinal axis within whose limits the flow velocity is maintained equal to the axial velocity is a distinctive feature of gradiant flows of viscoplastic fluids in pipes. It is necessary to recognize the existence in the core of small longitudinal deformations. This fact is confirmed by the very closed agreement of the theoretical solution of the development problem with the velocity of a viscoplastic fluid in the initial hydrodynamic segment of a circular pipe, carried out under our assumption, and experimental data [15]. Analogously [15], to determine the core radius we write the condition

$$\mathrm{I}\,F_2(\theta) \geqslant 8\varphi(\mathrm{I})\,(-dP/d\xi)\eta_0. \tag{5}$$

Equality will hold in Eq. (5) at the core boundary. The boundary conditions for the temperature and velocity components have the form

$$V_z = V_r' = 0 \quad \text{when} \quad \eta = 1; \tag{6}$$

$$V_z = V_{1z},\ \partial V_z/\partial\eta = 0 \ \text{when}\ \eta = \eta_0(\xi); \tag{7}$$

$$\partial\theta/\partial\eta = 0 \ \text{when}\ \eta = 0; \tag{8}$$

$$\theta = 1 \ \text{when}\ \xi = 0; \tag{9}$$

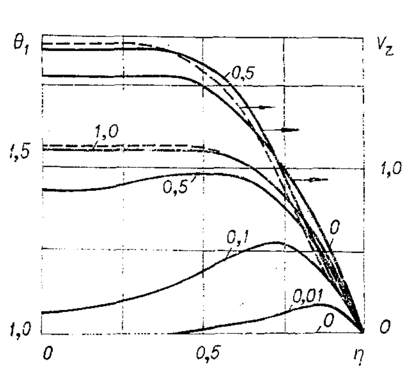

Fig. 1

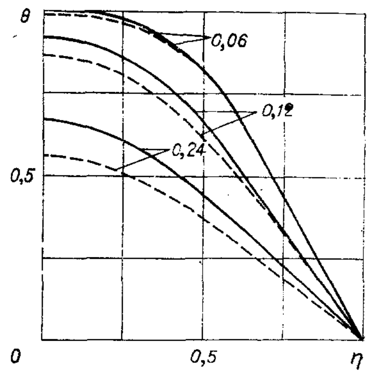

Fig. 2

$$\theta=0 \text{ when } \eta=1,\ \xi>0. \tag{10}$$

In Eq. (7) V_{1z} denotes core velocity. We add to Eqs. (6)-(10) the condition that fluid flow rate is constant,

$$V_{1z}\eta_0^2(\xi)+2\int\limits_{\eta_0(\xi)}^{1}\eta V_z d\eta=1. \tag{11}$$

We note that the third boundary condition (7) was transferred in [13] to the pipe axis; the motion and heat-conductivity equations were thus extended to the entire flow region without taking into account the fact that

$$\partial V_z/\partial\eta=0$$

within the core.

Here we must emphasize that the first two equations of the system (4) were written for the visco-plastic region of the flow.

The motion equation for the core reduces to

$$V_z=V_{1z},$$

the core velocity being found from the boundary condition (7), while the heat-conductivity equation can be extended to the entire flow region if we take into account that heat release of viscous friction is absent in the core.

With this remark in mind, we further simplify the system (4). We integrate the first equation of the system (4) over η taking into account Eq. (5) and the second boundary condition (7), and eliminate the derivative $\partial V_z/\partial\eta$ from the second equation of the system (4). These operations finally reduce Eqs. (4) to the simpler form

$$\frac{\partial V_z}{\partial\eta}=F_1^{-1}(\theta)\left[-4\varphi(\mathrm{I})\left(-\frac{dP}{d\xi}\right)\eta+\frac{\mathrm{I}}{2}F_2(\theta)\right],\quad 0\leqslant\eta\leqslant\eta_0, \tag{12}$$

$$V_z=V_{1z}(\xi),\ \eta_0\leqslant\eta\leqslant 1;$$

$$V_r'\frac{\partial\theta}{\partial\eta}+V_z\frac{\partial\theta}{\partial\xi}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)+\alpha\Phi(\mathrm{I},\eta),\quad 0\leqslant\eta\leqslant 1;$$

$$\partial(\eta V_r')/\partial\eta+\partial(\eta V_z)/\partial\xi=0,$$

where $\Phi(\mathrm{I},\eta)$ is the distribution function for heat courses of viscous friction,

$$\Phi(\mathrm{I},\eta)=\begin{cases}\dfrac{8\varphi(\mathrm{I})}{F_1(\theta)}\dfrac{dP}{d\xi}\eta\left[4\varphi(\mathrm{I})\dfrac{dP}{d\xi}\eta+\dfrac{\mathrm{I}}{2}F_2(\theta)\right], & 0\leqslant\eta\leqslant\eta_0\\ 0, & \eta_0\leqslant\eta\leqslant 1.\end{cases}$$

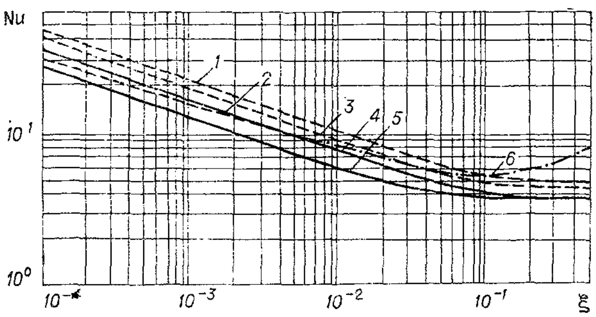

Fig. 3

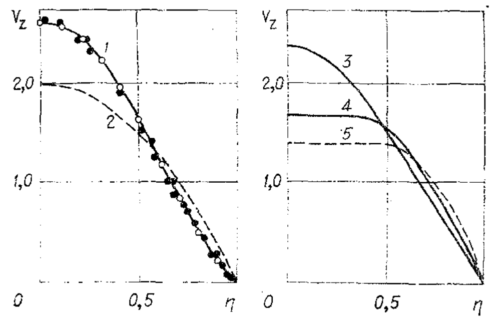

Fig. 4

Let us transform Eq. (11). Integrating the first equation of the system (12) and then substituting the resulting expression for V_z in Eq. (11) and integrating by parts, taking into account the first boundary condition (7), we obtain

$$\int_{\eta_0(\xi)}^{1} \eta^2 F_1^{-1}(\theta)\left[4\varphi(1)\left(-\frac{dP}{d\xi}\right)\eta - 1/2F_2(\theta)\right]d\eta = 1. \tag{13}$$

Equation (13) together with Eq. (5) determines the core radius if it is considered as a parameter.

The systems of equations (12), (5)-(10), and (13) were numerically solved on a computer using the method of finite differences with iterations.

A similar method was previously used for numerically solving a system of equations for a boundary layer in a compressible gas for longitudinal plate streamline [16].

Calculations were carried out for each case. In the first case, it was assumed that the rheological characteristics of the fluid depend on the temperature according to a hyperbolic law [17], and in the second case, according to an exponential law. We will first consider results of a numerical solution of the problem for the particular case when fluid temperature at the input and the temperature of the wall are identical and energy dissipation results from flow nonisothermicity. It is necessary here to vary the definition given above of dimensionless temperature.

Taking as a new dimensionless temperature the ratio

$$\theta_1 = T/T_W,$$

we may establish that a single variation, which it is necessary to carry out in the systems (12), (13) and (5)-(10), deals with the boundary condition (10) which is replaced by

$$\theta_1 = 1;\ \eta = 1;\ \xi > 0.$$

We assume that the rheological characteristics depend on temperature by a hyperbolic law [17], so that

$$F_1(\theta_1) = F_2(\theta_1) = \theta_1^{-1}.$$

Figure 1 illustrates the development of the temperature and velocity fields in a liquid ($I = 6$; $\alpha = 0.2$) from input to the pipe through the segment corresponding to their stabilization. The longitudinal coordinate ξ is the parameter indicated by the digits on the curve.

Distribution curves for temperature and velocity on the stabilization segment are denoted in Fig. 1 by a broken curve. A comparison of the broken curves and the exact solution of the problem obtained in [17] reveal that they completely coincide right through the third significant digit following the decimal point i.e., within the limits of error of the difference scheme [calculations were carried out with steps $\Delta\eta = \Delta\xi = 10^{-2}$ and the order of approximation of the difference scheme was $0(\Delta\eta^2)$ at $\partial\theta/\partial\xi = 0$].

Let us turn to a discussion of the results of a numerical solution of the systems (12), (5)-(10), and (13), assuming an exponential dependence of the rheological parameters of the fluid on temperature,

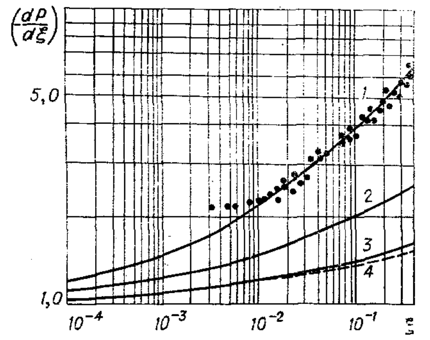

Fig. 5

$$f_1(T) = \exp\left[-\beta_1'(T - T_W)\right];$$
$$f_2(T) = \exp\left[-\beta_2'(T - T_W)\right];$$
$$\beta_i = \beta_i'(T_0 - T_W);\ F_i(\theta) = \exp(-\beta_i\theta),\ (i = 1, 2).$$

The temperature distribution with respect to radius and length of the pipe is illustrated in Fig. 2 (I = 11, $\alpha = 0$, and $\beta_1 = \beta_2 = 1$). The longitudinal coordinate ξ is the parameter of the curves. The precision of these results can be estimated by comparing the broken curves depicted in Fig. 2, which correspond to $\beta_1 = \beta_2 = 0$, to analogous curves given in [7] for these values of ξ and $\eta_0 = 0.5$ (I = 11 and core radius $\eta_0 = 0.495$). A comparison reveals that they totally coincide. It should be emphasized here that ξ is "stretched out" to twice that of the coordinate given in [7] by the same letter.

The dimensionless heat-transfer coefficient or Nusselt number, has the form

$$\mathrm{Nu} = 2\langle\theta\rangle^{-1}(d\theta/d\eta)_{\eta=1},$$

where $\langle\theta\rangle$ is the mean flow temperature,

$$\langle\theta\rangle = 2\int_0^1 \eta V_z(\eta)\,\theta(\eta)\,d\eta.$$

The variation of the Nusselt number along the pipe length as a function of the parameter I, β_1, β_2, and α is illustrated in Fig. 3.

(*1* — I = 38, $\beta_1 = \beta_2 = 0$, $\alpha = 0$;
2 — I = 0, $\beta_1 = 0$, $\alpha = 0$;
3 — I = 11, $\beta_1 = \beta_2 = 0$, $\alpha = 0$;
4 — I = 11, $\beta_1 = 1$, $\beta_2 = 0$, $\alpha = 0$;
5 — I = 11, $\beta_1 = \beta_2 = 1$, $\alpha = 0$;
6 — I = 11, $\beta_1 = 1$, $\beta_2 = 0$, $\alpha = 0.025$).

The nature of the variation of the curves in Fig. 3 indicates a decrease in fluid heat transfer if its rheological characteristics are temperature-dependent. The broken curves in Fig. 3 were calculated for constant rheological characteristics of the fluid. Curve 2, which corresponds to a viscous fluid (at I = 0) precisely coincides with the graph given in [2], and curve 3 precisely coincides with the graph depicting the variation of the Nusselt number for a viscoplastic fluid at $\eta_0 = 0.5$ and $\beta_1 = \beta_2 = 0$ [4, 7]. It is evident from Fig. 3 that energy dissipation, particularly at a distance from the input to the pipe, leads to a substantial increase in the Nusselt number. This fact has been studied in detail [4-7]. From the results of these investigations, the Nusselt parameter first monotonically decreases, then sharply increases at a significant distance from the input, and, finally attains stabilization. Figure 3 depicts two of these characteristic features. Stabilization apparently holds when $\xi > 0.5$, but no calculation was carried out for this region.

The velocity distribution across the pipe as a function of the parameters I and β_1 is illustrated in Fig. 4 (at $\xi = 0.034$ and $\alpha = 0$):

1 — I = 0, $\beta_1 = 1.36$; *2* — I = 0, $\beta_1 = 0$;
3 — I = 0, $\beta_1 = 1$; *4* — I = 11, $\beta_1 = 1$, $\beta_2 = 0$;
5 — I = 11, $\beta_1 = \beta_2 = 0$).

Curve 1 was drawn through points denoted by the light circles. They correspond to values of the velocity V_z found by a calculation at I = 0 and $\beta_1 = 1.36$. Experimental results [10] obtained in investigating the flow of a Newtonian fluid (glycerin) under these conditions ($\beta_1 = 1.36$) are indicated by dark circles. We note that the coordinate ξ is "stretched out" by four times that of the longitudinal x coordinate selected in [10]. The experimental and theoretical results can be seen to coincide well.

Variations in the velocity distributions along the pipe qualitatively explain these features of heat transfer and the increase in local temperatures for variable rheological characteristics of a fluid, since in this case an increase in the velocity of the central flow region facilitates more intensive heat transfer from the input to the pipe. This leads to a general increase in temperature and a decrease in heat transfer. Such a "gain" in heat losses is attained due to a sharp increase in the pressure drop.

A graphic representation of the most characteristic features in the variation of the pressure gradient along the pipe length is given by Fig. 5, which simultaneously illustrates a comparison of the experimental data [10] for a Newtonian fluid, indicated by circles, and numerical solutions:

$$(1 - I = 0,\ \beta_1=1.95,\ \alpha=0;$$
$$2 - I=0,\ \beta_1=1,\ \alpha=0;$$
$$3 - I=11,\ \beta_1=1,\ \beta_2=0,\ \alpha=0;$$
$$4 - I=11,\ \beta_1=1,\ \beta_2=0,\ \alpha=0.025).$$

Since, according to a previous bound [10], the maximal error in determining the pressure gradient amounts to 12 %, we may conclude that the experimental results and the numerical solution coincide to a high degree. At the same time, the systematic understatement of the calculated pressure gradient as compared to the experimental data for a region near the input pipe is evidently explained by the fact that effects related to the development of velocity in the initial hydrodynamic segment, which were estimated to be small, are of great value for this region.

It is well known that the length of the initial hydrodynamic segment is significantly less for a viscoplastic fluid than for a Newtonian fluid [3, 15]. We should therefore expect that the error noted above in the numerical solutions occurs in a highly narrow region directly at the input of the pipe and does not noticeably affect the precision of the calculation in the overwhelming majority of the cases.

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